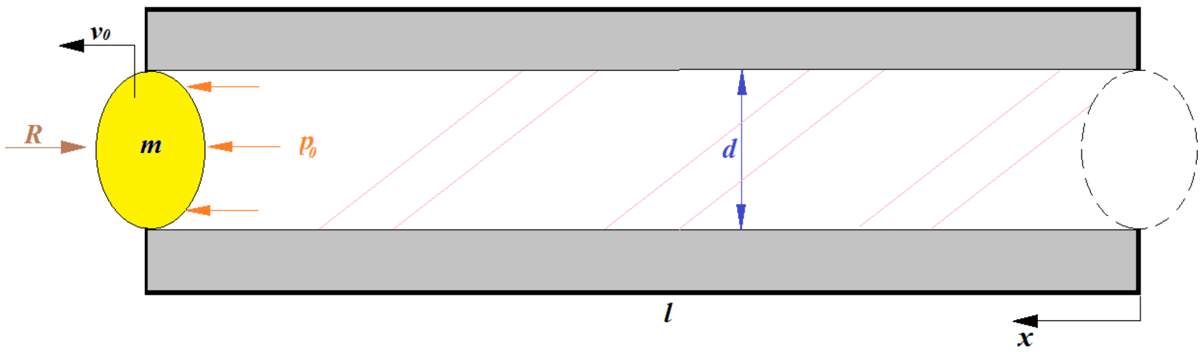


$R \neq 0$

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{x}}\right)-\frac{\partial E}{\partial x}=Q\ ,\ Q=F-R$$

$$E=\frac{1}{2}m\dot{x}^2+\frac{1}{2}I_0\dot{\varphi}^2\ ,\quad \begin{matrix}\varphi=2\pi sx\,[\mathrm{rad}]\\ [s]=\left[\frac{\mathrm{obr}}{\mathrm{m}}\right]\end{matrix}\quad \Rightarrow\ E=\frac{1}{2}m\dot{x}^2+2\pi^2 I_0 s^2\dot{x}^2=\left(\frac{1}{2}m+2\pi^2 I_0 s^2\right)\dot{x}^2$$

$$I_0\approx\frac{1}{2}m\left(\frac{d}{2}\right)^2\ \Rightarrow\ \frac{\partial E}{\partial \dot{x}}=m\left(1+\frac{1}{2}\pi^2d^2s^2\right)\dot{x}\ \Rightarrow\ \boxed{\frac{1}{2}m\left(2+\pi^2d^2s^2\right)\ddot{x}=F-R}\ \Rightarrow\ \frac{1}{2}m\left(2+\pi^2d^2s^2\right)v\frac{dv}{dx}=p\frac{\pi d^2}{4}-kv^2$$

$$\Rightarrow\ -\frac{m\left(2+\pi^2d^2s^2\right)}{4k}\frac{-2kv}{p\dfrac{\pi d^2}{4}-kv^2}dv=dx\,/\int(\ )\ \Leftrightarrow\ \ln\left|p\frac{\pi d^2}{4}-kv^2\right|=-\frac{4k}{m\left(2+\pi^2d^2s^2\right)}x+D$$

$$\Rightarrow\ p\frac{\pi d^2}{4}-kv^2=Ee^{-\frac{4k}{m\left(2+\pi^2d^2s^2\right)}x}\ \Rightarrow\ v(x,E)=\sqrt{\frac{1}{k}\left(p\frac{\pi d^2}{4}-Ee^{-\frac{4k}{m\left(2+\pi^2d^2s^2\right)}x}\right)}\ ,\ \text{W.P: } v(0,E)=\sqrt{\frac{1}{k}\left(p\frac{\pi d^2}{4}-Ee^0\right)}=0$$

$$\Rightarrow E=p\frac{\pi d^2}{4}\Rightarrow\ \boxed{v(x)=\sqrt{p\frac{\pi d^2}{4k}\left(1-e^{-\frac{4k}{m\left(2+\pi^2d^2s^2\right)}x}\right)}}\ ,\ \text{W.K: } v(x=l)=v_0=\sqrt{\bar{p}_0\frac{\pi d^2}{4k}\left(1-e^{-\frac{4k}{m\left(2+\pi^2d^2s^2\right)}l}\right)}$$

$$\Rightarrow\ \boxed{\bar{p}_0=10^{-6}\frac{4k}{\pi d^2}v_0^2\left(1-e^{\frac{-4kl}{m\left(2+\pi^2d^2s^2\right)}}\right)^{-1}[\mathrm{MPa}]}\ ,\ k=\frac{1}{2}\cdot C_x\cdot\rho\cdot A$$

$R=0$

$$\frac{1}{2}m\left(2+\pi^2d^2s^2\right)v\frac{dv}{dx}=F\ \Rightarrow\ \frac{1}{2}m\left(2+\pi^2d^2s^2\right)v\frac{dv}{dx}=p\frac{\pi d^2}{4}\ \Rightarrow\ vdv=p\frac{\pi d^2}{2m\left(2+\pi^2d^2s^2\right)}dx\,/\int(\ )$$

$$\Rightarrow\ v(x,D)=\sqrt{p\frac{\pi d^2}{m\left(2+\pi^2d^2s^2\right)}x+D}\ ,\ \text{W.P: } v(0,D)=0\ \Rightarrow\ D=0\ \Rightarrow\ \boxed{v(x)=\sqrt{p\frac{\pi d^2}{m\left(2+\pi^2d^2s^2\right)}x}}$$

$$\Rightarrow\ v(x=l)=v_0=\sqrt{\bar{p}_0\frac{\pi d^2 l}{m\left(2+\pi^2d^2s^2\right)}}\Rightarrow\ \boxed{\bar{p}_0=10^{-6}\frac{m\left(2+\pi^2d^2s^2\right)}{\pi d^2 l}v_0^2\,[\mathrm{MPa}]}$$

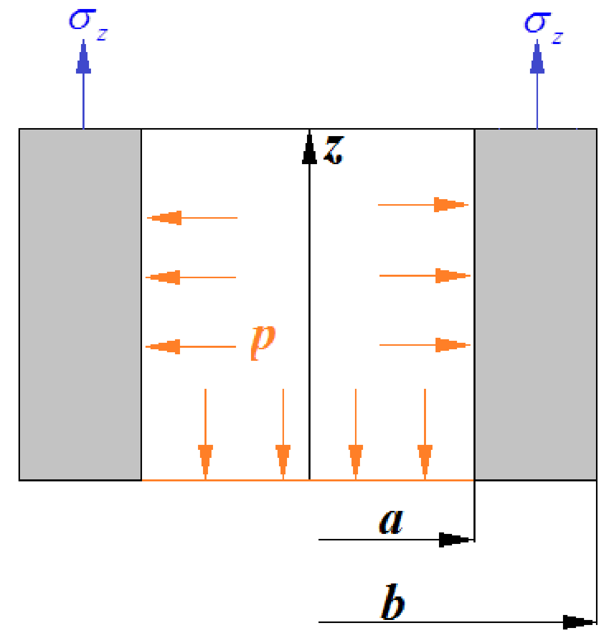
$$\boxed{\sigma_r(r) = A - \frac{B}{r^2}}\ ,\ \boxed{\sigma_\varphi(r) = A + \frac{B}{r^2}}$$

$$\begin{aligned} \sigma_r(a) = -p \ &\Leftrightarrow\ A - \frac{B}{a^2} = -p \quad /- \\ \sigma_r(b) = 0 \ &\Leftrightarrow\ A - \frac{B}{b^2} = 0 \end{aligned} \qquad \Rightarrow \qquad \begin{array}{l} \boxed{\sigma_r(r) = \dfrac{a^2}{b^2-a^2}\, p\left[1-\left(\dfrac{b}{r}\right)^2\right]} \\ \boxed{\sigma_\varphi(r) = \dfrac{a^2}{b^2-a^2}\, p\left[1+\left(\dfrac{b}{r}\right)^2\right]} \end{array}$$

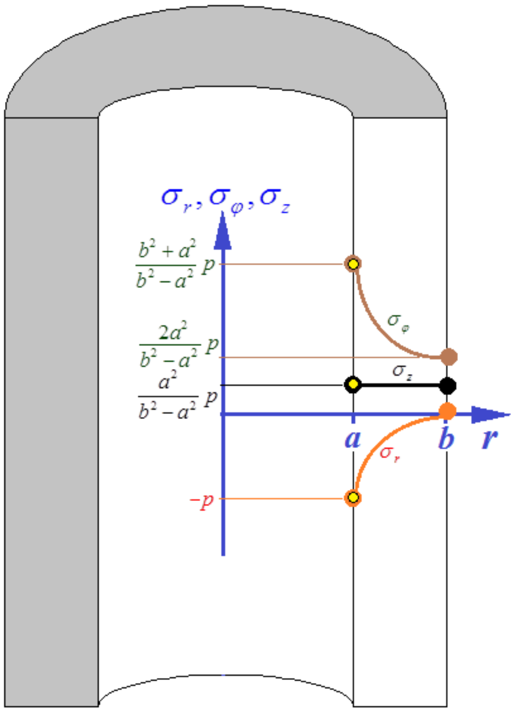
$$\to \frac{B}{a^2} - \frac{B}{b^2} = p \ \Rightarrow\ \underline{B = \frac{a^2 b^2}{b^2-a^2}\, p}\Big| \ ,\ A = \frac{B}{b^2} \to\ \underline{A = \frac{a^2}{b^2-a^2}\, p}\Big|$$

$$\sigma_\varphi(a) = \frac{a^2}{b^2-a^2}\, p\left[1+\left(\frac{b}{a}\right)^2\right] = \frac{b^2+a^2}{b^2-a^2}\, p$$

$$\sigma_\varphi(b) = \frac{a^2}{b^2-a^2}\, p\left[1+\left(\frac{b}{b}\right)^2\right] = \frac{2a^2}{b^2-a^2}\, p$$



$$\sum_i F_{iz} = 0 \ \Rightarrow\ \sigma_z \pi\left(b^2-a^2\right) - p\pi a^2 = 0 \ \Rightarrow\ \sigma_z = \frac{a^2}{b^2-a^2}\, p = \frac{\sigma_r + \sigma_\varphi}{2}$$



$$\sigma_I = \sigma_\varphi \quad \sigma_{II} = \sigma_z \quad \sigma_{III} = \sigma_r$$

$$\boxed{\sigma_{red\,\max}^{HMH} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2}} =$$

$$= \frac{1}{\sqrt{2}}\sqrt{\left(\frac{b^2+a^2}{b^2-a^2}p - \frac{a^2}{b^2-a^2}p\right)^2 + \left(\frac{a^2}{b^2-a^2}p - (-p)\right)^2 + \left(-p - \frac{b^2+a^2}{b^2-a^2}p\right)^2} = \sqrt{3}\frac{b^2}{b^2-a^2}p = \sqrt{3}\frac{\left(\frac{b}{a}\right)^2}{\left(\frac{b}{a}\right)^2 - 1}p$$

$$\sigma_{red\,\max}^{HMH} \le k_r \quad \Leftrightarrow \quad \sqrt{3}\frac{\left(\frac{b}{a}\right)^2}{\left(\frac{b}{a}\right)^2 - 1}p \le k_r \Rightarrow \left(\frac{b}{a}\right)^2 - \frac{1}{1-\sqrt{3}\frac{p}{k_r}} \ge 0$$

$$\left(\frac{b}{a} - \frac{1}{\sqrt{1-\sqrt{3}\frac{p}{k_r}}}\right)\left(\frac{b}{a} + \frac{1}{\sqrt{1-\sqrt{3}\frac{p}{k_r}}}\right) \ge 0 \Rightarrow \frac{b}{a} \ge \frac{1}{\sqrt{1-\sqrt{3}\frac{p}{k_r}}} \Rightarrow b \ge \frac{a}{\sqrt{1-\sqrt{3}\frac{p}{k_r}}}, \quad a = \frac{d}{2}, \; p = \bar{p}_0$$

$$\Rightarrow b \ge \frac{d}{2\sqrt{1-\sqrt{3}\frac{\bar{p}_0}{k_r}}} \Rightarrow \bar{h} = b - a \ge \frac{d}{2}\left(\frac{1}{\sqrt{1-\sqrt{3}\frac{\bar{p}_0}{k_r}}} - 1\right) \Rightarrow \bar{h} \ge \frac{d}{2}\left(\frac{1}{\sqrt{1-\sqrt{3}\frac{\bar{p}_0}{k_r}}} - 1\right) \Leftrightarrow k_r > \sqrt{3}\bar{p}_0$$

$$k \neq 0 \Rightarrow \boxed{\bar{h} \geq \frac{d}{2}\left\{\left[1-4\sqrt{3}\frac{kv_0^2}{\pi k_r d^2\left(1-e^{\frac{-4kl}{m\left(2+\pi^2 d^2 s^2\right)}}\right)}\right]^{-\frac{1}{2}}-1\right\}}$$

$$k = 0 \Rightarrow \boxed{\bar{h} \geq \frac{d}{2}\left[\left(1-\sqrt{3}\frac{2+\pi^2 d^2 s^2}{\pi k_r d^2 l}mv_0^2\right)^{-\frac{1}{2}}-1\right]}$$

$$\bar{p}_0\left(.308\text{Win}\,,\,k\neq 0\right)=\frac{4\cdot 7.468948\cdot 10^{-6}}{\pi\left(0.00782\right)^2}804^2\left(1-e^{\frac{-4\cdot 7.468948\cdot 10^{-6}\cdot 0.62}{0.0089\left(2+\pi^2\left(0.00782\right)^2 4.84^2\right)}}\right)^{-1}=97.65\left[\text{MPa}\right]$$

$$\bar{p}_0\left(.308\text{Win}\,,\,k=0\right)=10^{-6}\frac{0.0089\left(2+\pi^2\cdot 0.00782^2\cdot 4.84^2\right)\cdot 804^2}{\pi\cdot\left(0.00782\right)^2 0.62}=97.28\left[\text{MPa}\right]$$

$$\bar{p}_0\left(.308\text{Win}\,,\,k=0\;,\;s=0\right)=10^{-6}\frac{0.0089\cdot 2\cdot 804^2}{\pi\cdot\left(0.00782\right)^2 0.62}=96.60\left[\text{MPa}\right]$$

$$\boxed{\bar{p}_0=10^{-6}\cdot\frac{2mv_0^2}{\pi l d^2}}\left[\text{MPa}\right] \qquad \boxed{p_{\max}=3\cdot\bar{p}_0}\left[\text{MPa}\right]$$

$$\boxed{h \geq 10^3\cdot\frac{d}{2}\left[\left(1-\sqrt{3}\frac{p_{\max}}{k_r}\right)^{-\frac{1}{2}}-1\right]}\left[\text{mm}\right] \Leftrightarrow k_r > \sqrt{3}p_{\max}$$

$d\left[\text{m}\right] \rightarrow$ kaliber

$m\left[\text{kg}\right] \rightarrow$ masa pocisku

$l\left[\text{m}\right] \rightarrow$ dlugość lufy

$v_0\left[\frac{\text{m}}{\text{s}}\right] \rightarrow$ prędkość na wylocie z lufy

$k_r\left[\text{MPa}\right] \rightarrow$ naprężenia dopuszczalne na rozciąganie

$h\left[\text{mm}\right] \rightarrow$ grubość scianki

dla : $d=0.44"\quad l=5.5"\quad m=0.0089\left[\text{kg}\right]\quad v_0=300\left[\frac{\text{m}}{\text{s}}\right]$

$$\bar{p}_0\left(.44CP\right)=10^{-6}\frac{2\cdot 0.0089\cdot 300^2}{\pi\cdot 0.1397\cdot\left(0.01143\right)^2}=27.94\left[\text{MPa}\right] \Rightarrow p_{\max}=3\cdot 27.94=83.82\left[\text{MPa}\right]$$

$$\Rightarrow k_r > \sqrt{3}\cdot 83.82\left[\text{MPa}\right]\;,\;k_r := 300\left[\text{MPa}\right]$$

$$h\left(.44CP\right)\geq 10^3\cdot\frac{0.01143}{2}\cdot\left[\left(1-\sqrt{3}\frac{83.82}{300}\right)^{-\frac{1}{2}}-1\right]=2.24\left[\text{mm}\right]$$